Identification and Control with Gaussian Processes with formal guarantees


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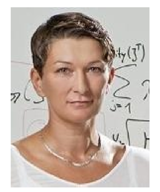
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Motivation

Precise and safe control for real-world systems is challenging due to uncertain system models and external disturbances

[Soft Robotics Inc] [KUKA] [DARPA] [Schaeffler]

Idea: Gaussian process regression for identification and control

Challenge: Providing formal guarantees for stable and safe operation

Research Statement

Safe, efficient and rational use of machine learning in control

Recent research

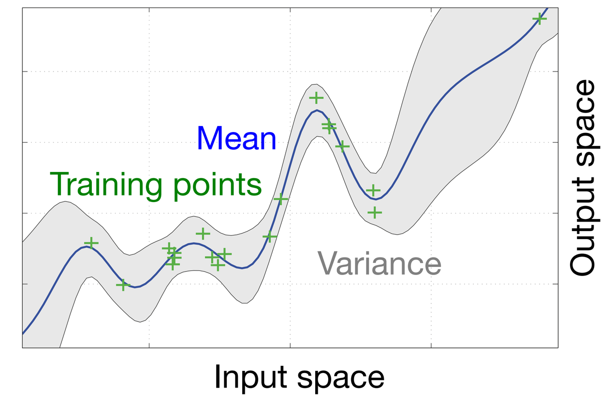
- Stability of data-driven systems
- Model error estimation for efficient learning
- Integration of prior knowledge for hybrid learning approaches

Future research

- Holistic learning including task, system and controller
- Exploiting the properties of data-driven models for smarter control

Gaussian Process Models

- Flexible nonparametric regression
- Based on Bayesian probability
- Noisy training data
- Explicit uncertainty description

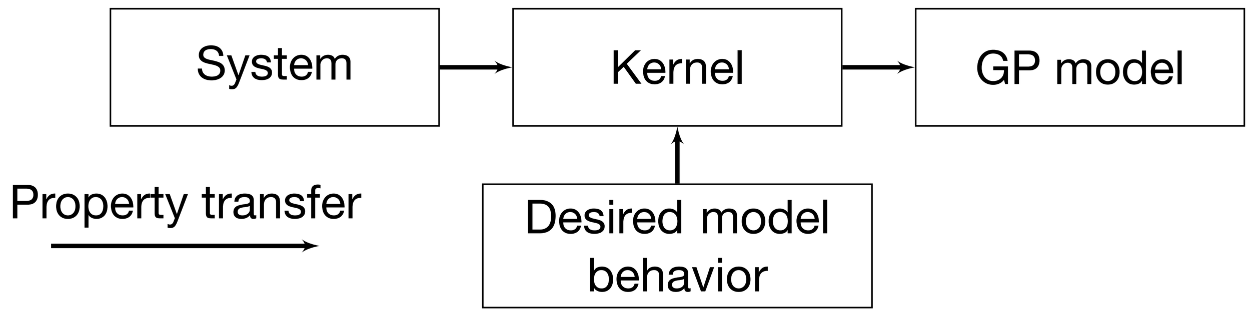


$$\mu(y^*|\boldsymbol{x}^*, \mathcal{D}) = \boldsymbol{k}_\varphi^\top(\boldsymbol{x}^*, X)(K_\varphi(X,X) + I\sigma_n^2)^{-1}Y$$
$$\Sigma(y^*|\boldsymbol{x}^*, \mathcal{D}) = k_\varphi(\boldsymbol{x}^*, \boldsymbol{x}^*) - \boldsymbol{k}_\varphi^\top(\boldsymbol{x}^*, X)(K_\varphi(X,X) + I\sigma_n^2)^{-1}\boldsymbol{k}_\varphi(\boldsymbol{x}^*, X)$$

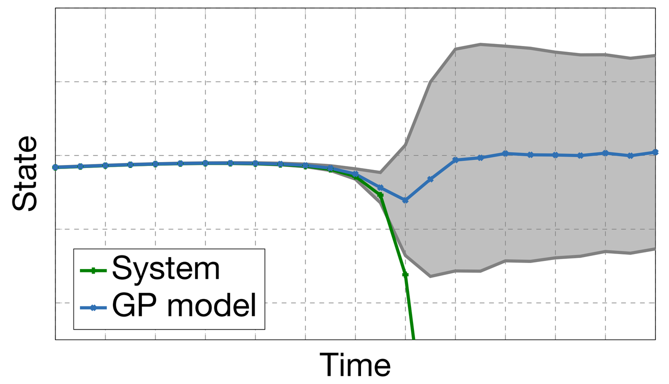
Identification

Rational kernel selection

- Kernel constrains the capabilities of the model
- Active injection of properties



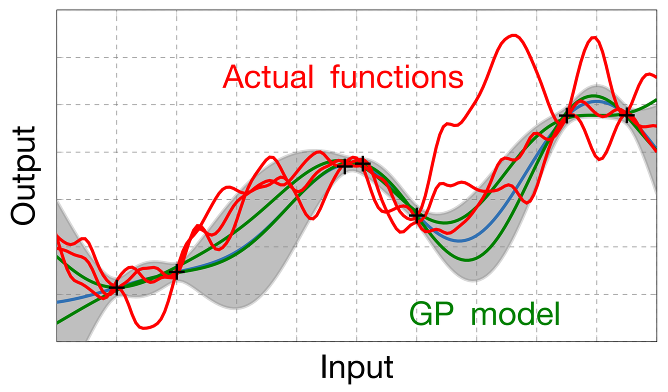
Squared exponential kernel generates always bounded systems



Model error

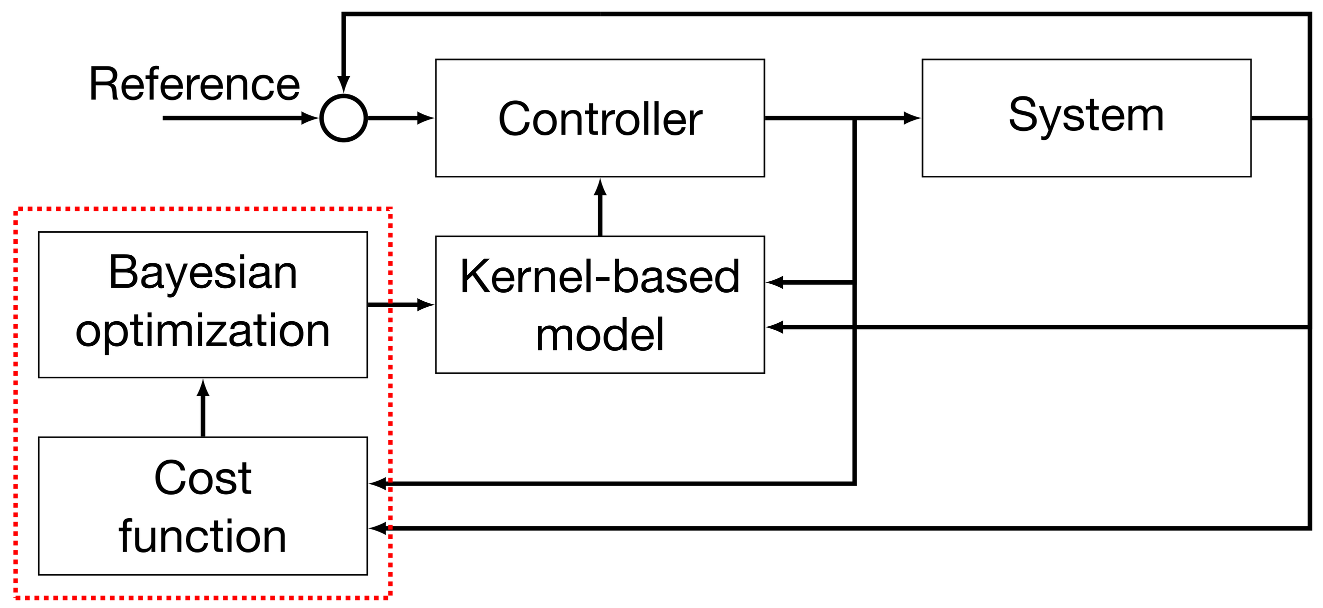
Misspecified GPs underestimate the model error

⇒ Derivation of an upper bound for prediction error



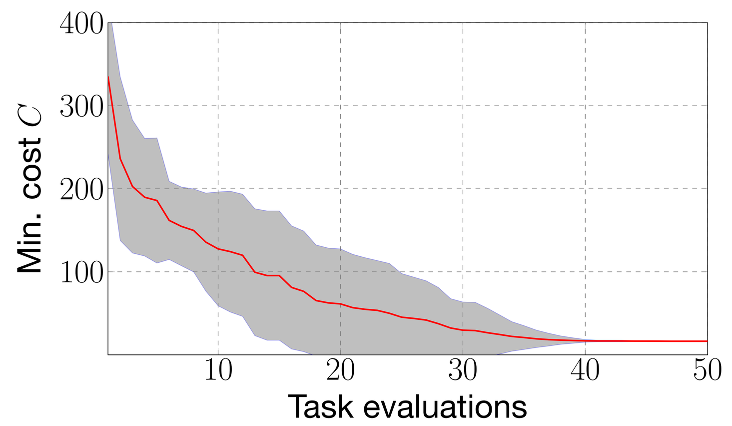
Hyperparameter optimization

Including the task into the optimization process



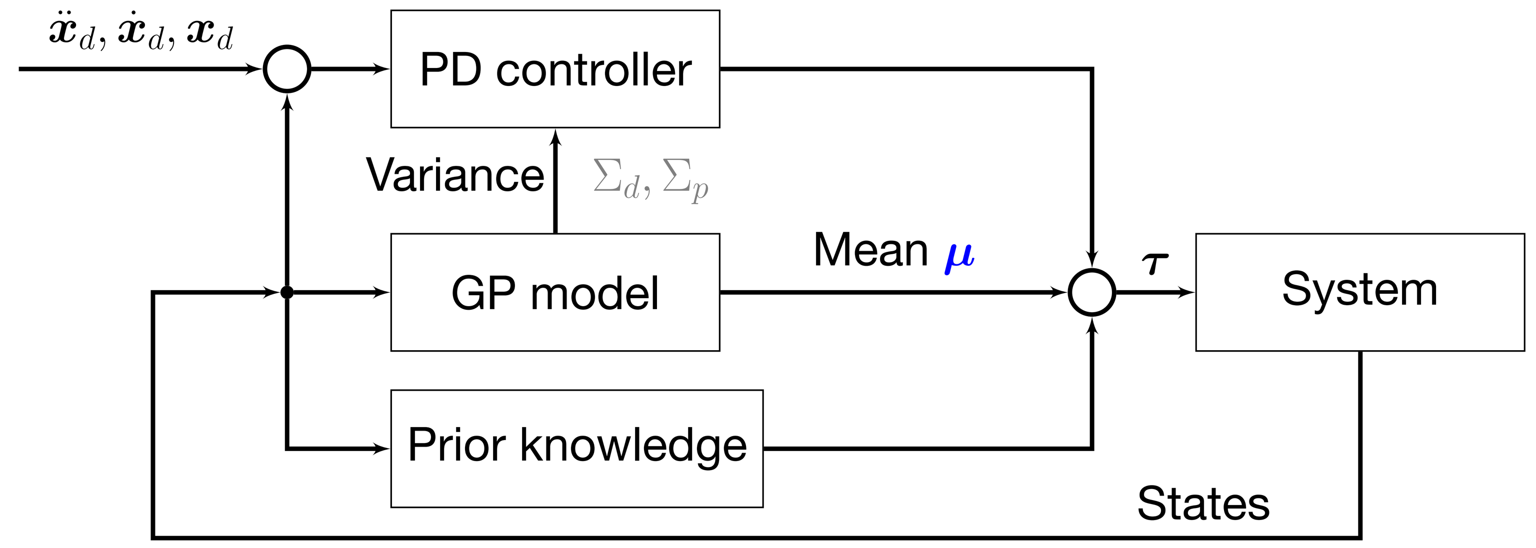
- Classical approaches focus on data-based optimization only
- Closed-loop performance gives valuable information for the optimization
- ⇒ More comprehensive interpretation of the data
- ⇒ Stability can be preserved

Task-based hyperparameter tuning using Bayesian optimization improves the closed-loop performance



Control

Gaussian process based computed torque control



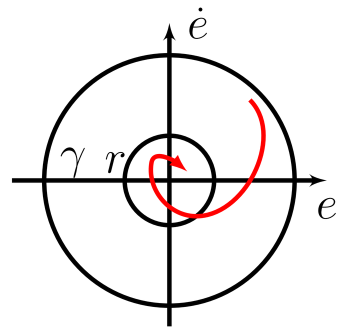
Adaptive feedback gains based on the model uncertainty

$$\boldsymbol{\tau} = \underbrace{\text{Prior knowledge} + \boldsymbol{\mu}(\tilde{\boldsymbol{\tau}}|\ddot{\boldsymbol{x}}, \dot{\boldsymbol{x}}, \boldsymbol{x}, \mathcal{D})}_{\text{Feedforward}} \underbrace{- K_d(\Sigma_d(\tilde{\boldsymbol{\tau}}|\dot{\boldsymbol{x}}, \boldsymbol{x}, \mathcal{D}))\dot{\boldsymbol{e}} - K_p(\Sigma_p(\tilde{\boldsymbol{\tau}}|\boldsymbol{x}, \mathcal{D}))\boldsymbol{e}}_{\text{Feedback}}$$

Stability analysis

The tracking error of the closed loop is uniformly ultimately bounded

Tracking error $r \sim \frac{\text{GP model error}}{\|\text{Feedback gain functions}\|}$



Improved performance inside and outside the training data set

Safe interaction due to low feedback gains

Selected Publications

T. Beckers, D. Kulic, and S. Hirche. Stable Gaussian process based tracking control of Euler-Lagrange systems. *Automatica*, 103:390–397, 2019.

T. Beckers, J. Umlauft, and S. Hirche. Mean square prediction error of misspecified Gaussian process models. In *Proc. of the Conference on Decision and Control*, 2018.

T. Beckers and S. Hirche. Passive rendering of a class of nonlinear systems with Gaussian process models. In *Proc. of the European Control Conference*, 2018.

T. Beckers, J. Umlauft, D. Kulić, and S. Hirche. Stable Gaussian process based tracking control of Lagrangian systems. In *Proc. of the Conference on Decision and Control*, 2017.

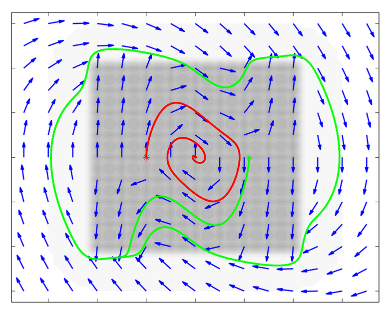
T. Beckers, J. Umlauft, and S. Hirche. Stable model-based control with Gaussian process regression for robot manipulators. In *Proc. of the 20th IFAC World Congress*, 2017.

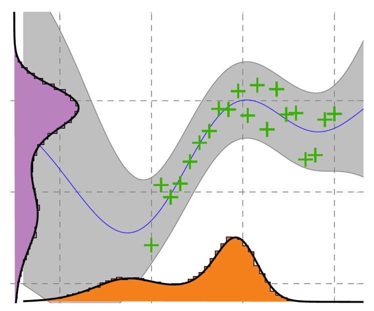
T. Beckers and S. Hirche. Stability of Gaussian process state space models. In *Proc. of the European Control Conference*, 2016.

T. Beckers and S. Hirche. Equilibrium distributions and stability analysis of Gaussian process state space models. In *Proc. of the Conference on Decision and Control*, 2016.

Evaluation

- Simulation of stable learning
- Property injection analysis
- Learning of robot dynamics





From simulation to application in real-world scenarios

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