Setting - Unknown LTI System

![Diagram of a single-input single-output (SISO) system with a gain G.]  

- Control of a linear time-invariant (LTI) system $G$ of order $n$, directly from measured data sequences $\{u_k, y_k\}_{k=0}^\infty$ without prior system identification.
- Goal: Tracking of a setpoint $(\bar{u}, \bar{y})$.
- Method: Data-driven model predictive control (MPC).
- Theoretical guarantees with noisy output measurements $\Rightarrow$ robust data-driven MPC.

Trajectory-based System Representation

For a sequence $\{u_k\}_{k=0}^\infty$ define the Hankel matrix

$$H_k(u) = \begin{bmatrix} u_0 & u_1 & \ldots & u_{k-1} \\ u_1 & u_2 & \ldots & u_k \\ \vdots & \vdots & \ddots & \vdots \\ u_{k-1} & u_{k-2} & \ldots & u_0 \end{bmatrix}.$$  

Definition: A signal $\{u_k, y_k\}_{k=0}^\infty$ with $u_k \in \mathbb{R}^n$ is persistently exciting of order $L$ if and only if there exists $\alpha \in \mathbb{R}^{n \times L+1}$ such that

$$H_k(u) \alpha = [u],$$

(1)

Implications
- Time-shifts of measured data span all other trajectories.
- Model-free, data-driven system parametrization.
- Only requirements: persistence of excitation & upper bound on model order.
- Has been employed to verify dissipativity [3] and to design stabilizing controllers [4].

Robust Data-Driven Model Predictive Control

Realistic four tank system [6]:
- Stable, 4 states, 2 inputs, 2 outputs, noisy measurements with $N = 400$, $\varepsilon = 0.005$.
- No model knowledge.
- Design parameters: $L = 30$, $Q = I$, $R = 5 \cdot 10^{-7} I$, $\lambda = 1000$, $\lambda = 0.1$.
- Goal: Tracking of a desired setpoint $(\bar{u}, \bar{y})$.

**Example**

![Graph showing closed-loop output $y$ and MPC iteration.]

Confirms theoretical results:
- The proposed scheme tracks the desired setpoint & the tracking error increases with $\varepsilon$.
- Inadequate tuning of $\lambda_\alpha$, $\lambda_\gamma \Rightarrow$ unstable closed loop.
- The scheme without terminal constraints [5] destabilizes the system.

**Conclusion**
- First results on data-driven (model-free) MPC with stability and robustness guarantees.
- Connection between system & design parameters and closed-loop region of attraction.
- Theoretical results confirmed in practical example.

**References**