Learning Turbulent Flows using Mobile Robots

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INTRODUCTION

Objective: model-based learning of turbulent flows using empirical data collected by mobile robots to improve on numerical solutions obtained from Reynolds-Averaged Navier Stokes (RANS) models.

- numerical methods: less expensive but require experimental validation
- data-driven methods: reliable but require measurement at all points

Model-based learning: combine numerical solutions with empirical data using Bayesian inference framework

RANS MODELS

Turbulent flow is characterized by random fluctuations in the flow properties. In engineering applications we are interested in averaged properties of the flow, e.g., the mean velocity components, and not the instantaneous fluctuations. These averaged properties can be obtained using RANS models:

- Ideally ensemble average:

$$\hat{\mathbf{q}}(x,t) = \frac{1}{\hat{n}} \sum_{j=1}^{\hat{n}} \mathbf{q}^j(x,t)$$

- Instead decompose velocity vector as

$$\mathbf{q}(x,t) = \mathbf{q}(x) + \epsilon(x,t)$$

where time-averaged velocity is

$$\mathbf{q}(x) = \lim_{T\to\infty} \frac{1}{T} \int_{t_1}^{t_1+T} \mathbf{q}(x,t)dt$$

Assumptions:

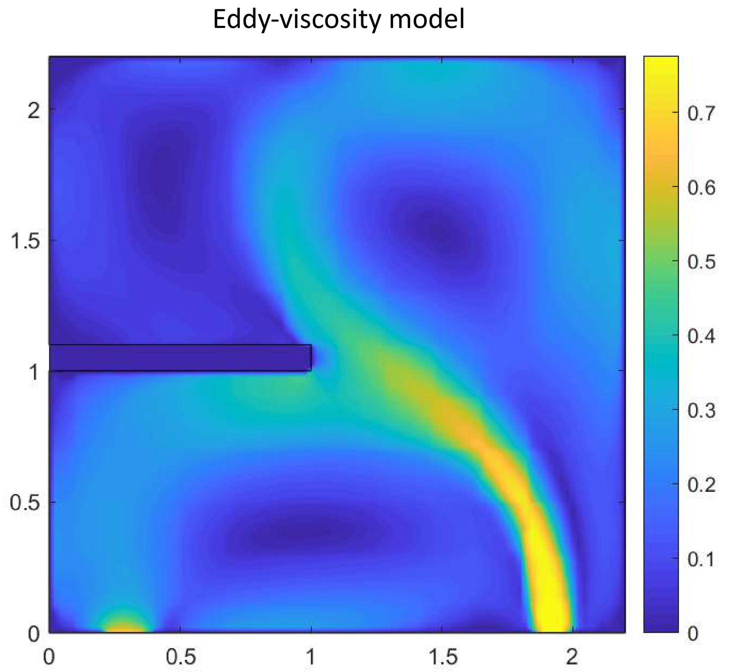
- statistical steadiness: $q(t)$ independent of t_1
- ergodicity: $\hat{\mathbf{q}}(x,t) = \mathbf{q}(x)$

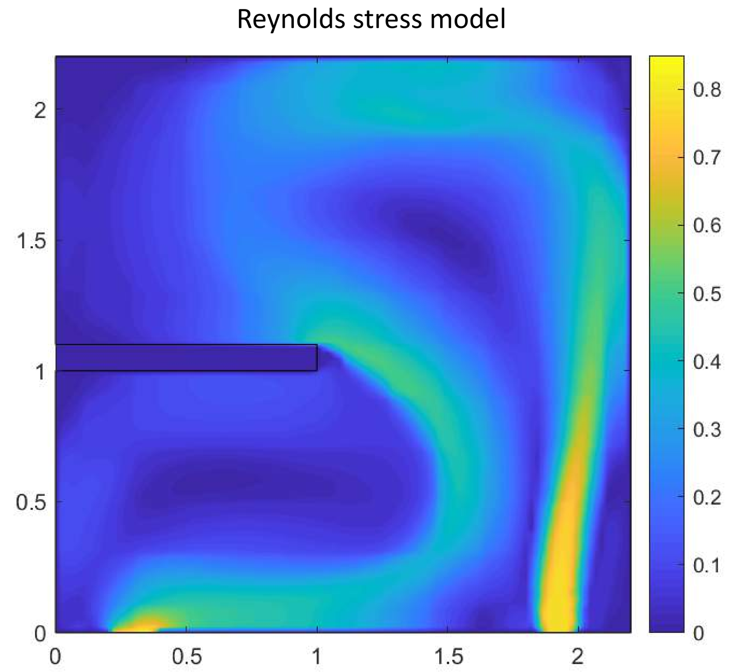
Closure problem:

velocity decomposition → extra unknowns → multiple RANS models

Challenges:

- In general, solutions provided by different RANS models are incompatible with each other and more importantly with the real world.

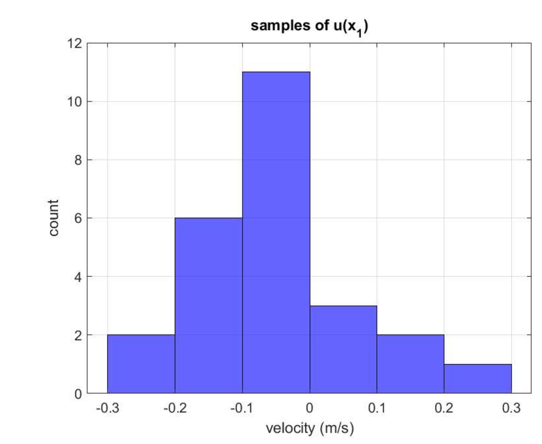


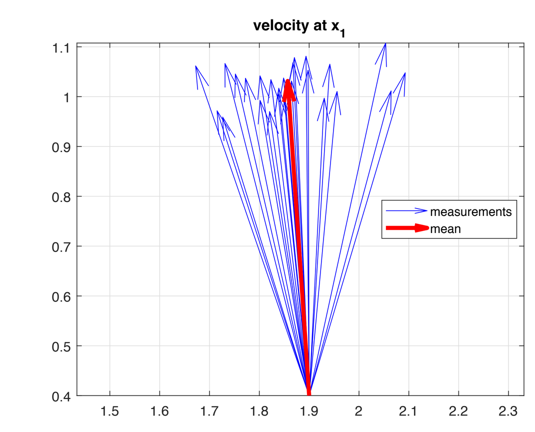


- Precise knowledge of boundary conditions and domain geometry is unavailable.
- Appropriate meshing is necessary to properly capture the boundary layer and get convergent solutions from RANS models.

MODEL-BASED LEARNING

Time-averaged properties are Normally distributed regardless of the underlying distribution of the instantaneous velocity field:





Given a numerical solution, **Gaussian process** $\mathcal{GP}(\mu(x), \bar{\kappa}(x,x'))$ models the mean velocity components:

- prior mean $\mu(x)$ is given by the numerical solution
- kernel $\bar{\kappa}(x,x') = \bar{\sigma}^2 \rho(x,x')$ where $\rho(x,x') = \left(1 - \frac{\|x-x'\|}{\ell}\right)_+^2$

Measurement model: $y(x) \sim \mathcal{GP}(\mu(x), \kappa(x,x'))$ with additive Gaussian noise $\epsilon \sim N(0,\sigma^2(x))$ such that $\kappa(x,x') = \bar{\kappa}(x,x') + \sigma^2(x)\,\delta(x-x')$.

Problem formulation:

- $\bar{n}$ models for combination of RANS models and boundary conditions
- Discrete distribution over models: $\tilde{\pi}(\mathcal{M}_j) = \{p_j, \mathcal{M}_j\}_{j=1}^{\bar{n}}$
- A Gaussian process for each model: $\mathcal{GP}(\mu(x|\mathcal{M}_j), \bar{\kappa}(x,x'|\mathcal{M}_j))$

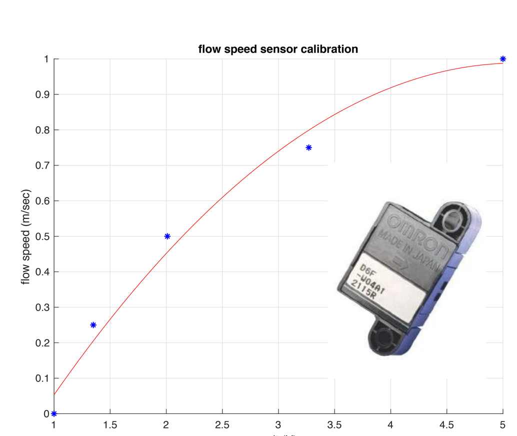
Given a set of empirical measurements at locations X, find

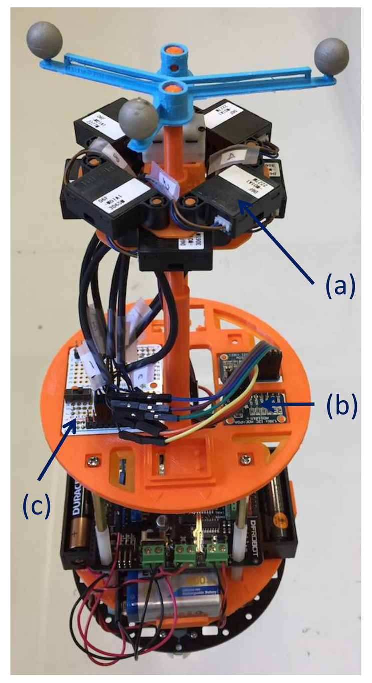
- posterior probability of models $p_j = \tilde{\pi}(\mathcal{M}_j|\mathcal{X})$
- posterior distribution given each model $\mathcal{N}(\mu(x|\mathcal{X},\mathcal{M}_j), \gamma^2(x|\mathcal{X},\mathcal{M}_j))$

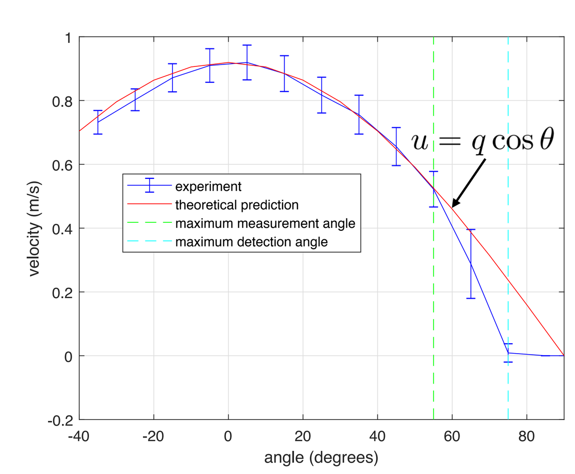
Then, the posterior distributions of flow properties are **Gaussian mixtures**.

MOBILE ROBOT

Sensor Type	Air Velocity Sensor
Range	0m/s to 1m/s
Accuracy:	+/- 5% full scale
Operating Voltage:	10.8V to 26.4V
Output Voltage:	1V to 5V







a) flow sensor set
b) analog to digital converter
c) power breadboard

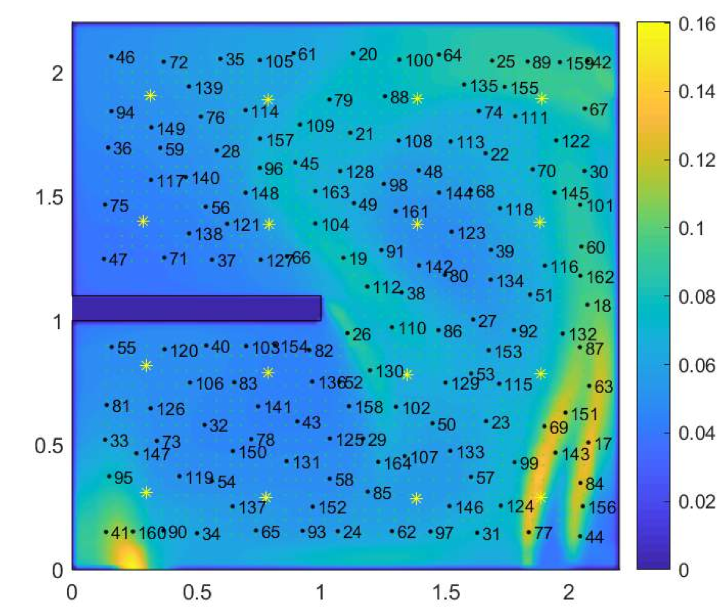
Planning: given a set of candidate measurement locations S and model probabilities $p_{j,k} = \tilde{\pi}(\mathcal{M}_j|\mathcal{X}_k)$, determine the next measurement location by

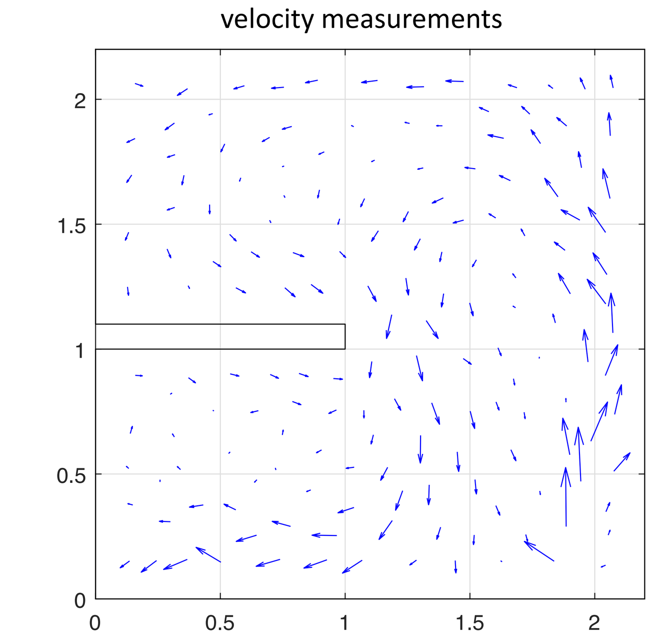
$$x_{k+1}^* = \text{argmax}_{x\in\mathcal{S}\setminus\mathcal{X}_k} \sum_{j=1}^{\bar{n}} p_{j,k}\,\gamma_u^2(x\,|\,\mathcal{X}_k, \mathcal{M}_j)$$

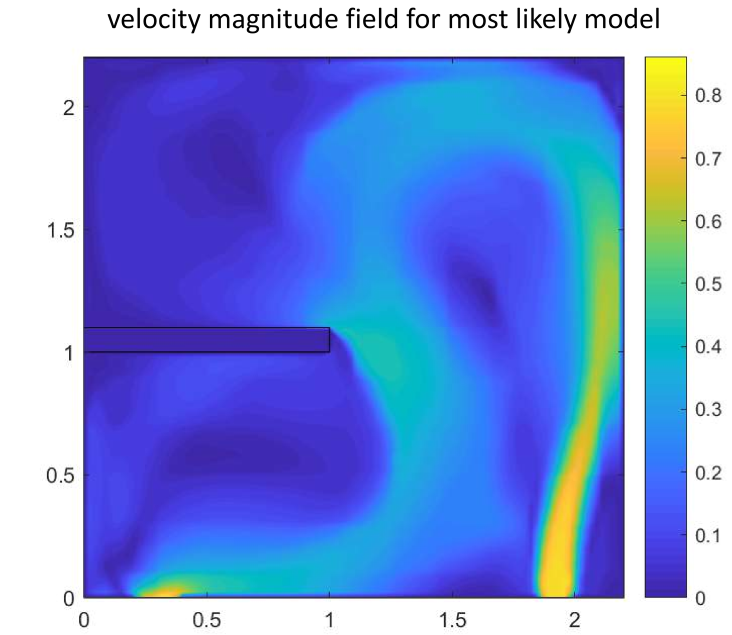
EXPERIMENTAL VALIDATION

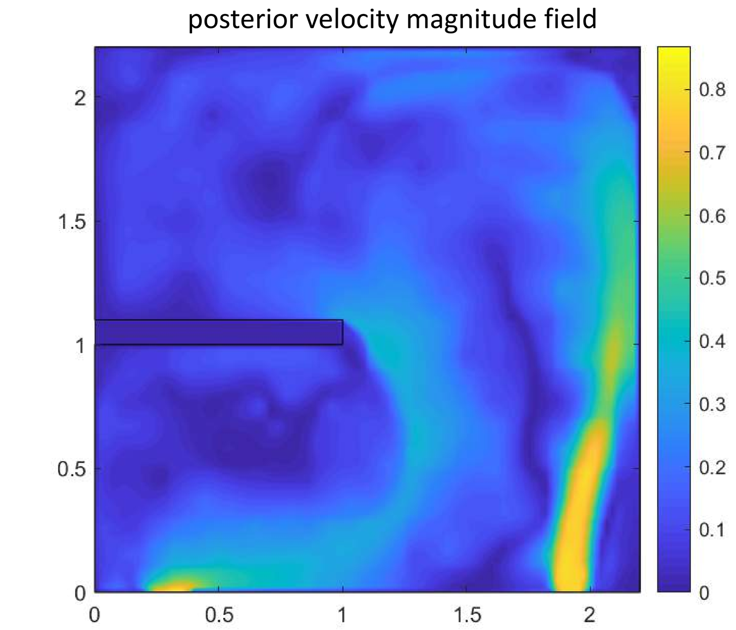
- average inlet velocity q_{in} = 0.78m/s
- $\bar{n}$ = 12 models for different combinations of BCs and RANS models
- $\bar{m}$ = 16 initial exploration measurements
- m = 200 maximum number of measurements
- $|S|$=1206 candidate measurement locations

Inference:



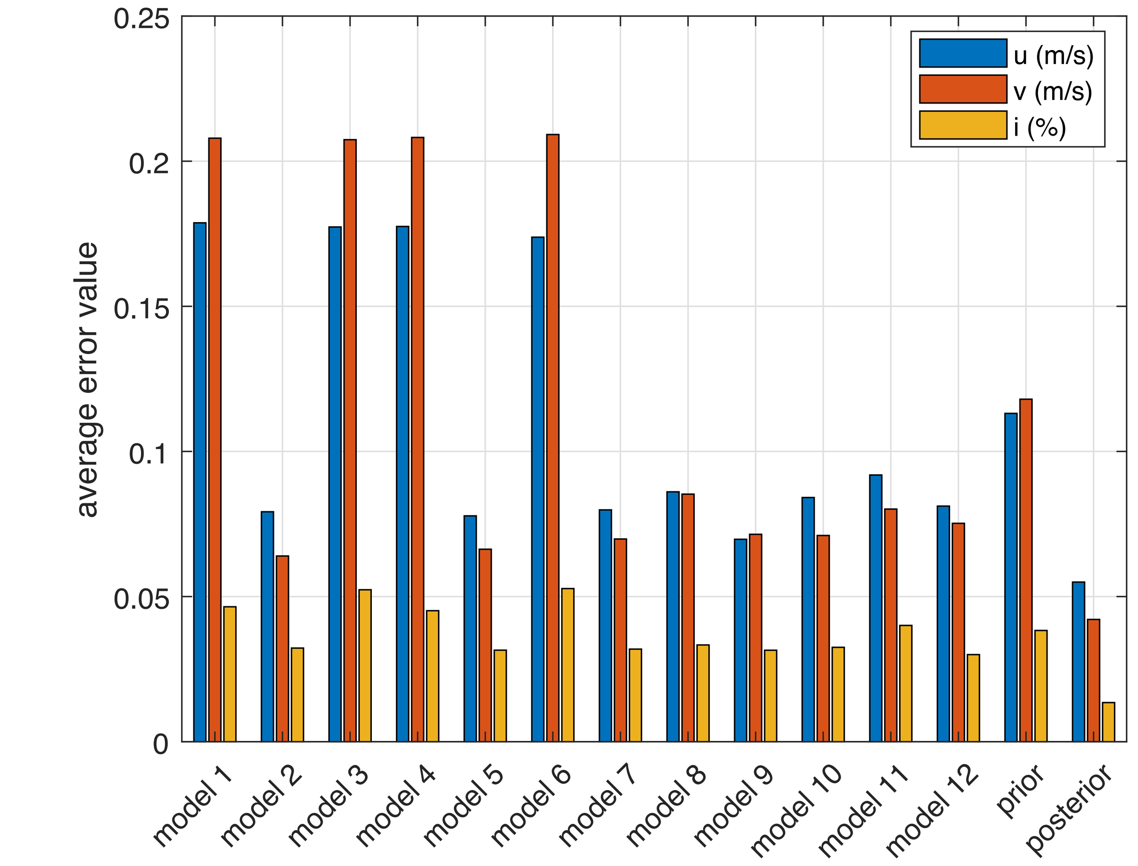






Prediction: 20 random measurement locations

- e_0 = 0.090m/s
- e_{164} = 0.037m/s (59% improvement)



REFERENCES

- R. Khodayi-mehr and M. M. Zavlanos*, "Model-based learning of turbulent flows using mobile robots," IEEE Transactions on Robotics, 2019.

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